# Fatigue damage of low amplitude cycles in low carbon steel

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Abstract The influences of low load cycles on fatigue damage in 0.15% C steel (C15E, No. 1.1141) are investigated in the very high cycle fatigue regime using ultrasonic fatigue testing equipment. Constant amplitude (CA) endurance limits at limiting lifetime of  $10<sup>9</sup>$  cycles are determined in cyclic tension–compression and cyclic torsion tests. Non-propagating fatigue cracks are found in specimens subjected to cyclic torsion loading at the endurance limit. The endurance limit is considered as maximum stress amplitude where possibly initiated fatigue cracks do not propagate to failure. Two-step variable amplitude (VA) tension–compression endurance tests are performed with repeat sequences consisting of high stress amplitudes above the endurance limit and far greater number of cycles below. The measured lifetimes are compared with linear damage accumulation calculations (Miner calculations). If the high stress amplitude is more than approximately 13% above the CA endurance limit, detrimental influences of low load cycles and failures at low damage sums are found. If the high stress is less than 13% above the CA endurance limit, numerous low load cycles cause prolonged fatigue lifetimes and specimens can sustain large damage sums without failure. Two-step VA fatigue crack growth investigations show that load cycles below the threshold stress intensity accelerate crack growth, if the high stress intensity is 18% or more above the CA threshold stress intensity. In repeat sequences with high stress intensities 14% above threshold stress intensity,

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low load cycles decelerated and stopped fatigue crack growth. Low load cycles can reduce or prolong fatigue lifetimes of low carbon steel and one reason is the accelerated or retarded fatigue crack growth due to numerous low amplitudes, and the maximum load amplitude of a VA load sequence determines whether detrimental or beneficial effects prevail.

## Introduction

Fatigue loading of engineering structures is predominantly under statistically varying loading conditions with relatively few high loads and much larger numbers of low load cycles. When components are stressed in the high cycle fatigue (HCF) or very high cycle fatigue (VHCF) regime, most load cycles in realistic in-service loading sequences are at stress amplitudes, which are too low to cause failure under constant amplitude loading conditions. Fatigue tests under variable amplitude loading conditions using different cumulative frequency distributions of stress amplitudes show failures in the VHCF regime (i.e., in the regime of  $10<sup>9</sup>$  cycles), when more than 99% of the stress amplitudes are below the mean endurance limit at  $10^9$  cycles [\[1](#page-10-0)]. Considering long fatigue lifetimes, possible influences of low load cycles on fatigue damage are of great interest therefore.

In the present investigation, fatigue damage in 0.15% C steel caused by numerous low load cycles is studied. Carbon steels without heat treatment show a pronounced change of slope of the S-N curve in the regime  $10^6$ - $10<sup>7</sup>$  cycles. Above these numbers of cycles, the probability for failure under constant stress amplitude loading is significantly reduced, and the endurance limit at limiting lifetime of  $10<sup>9</sup>$  cycles is hardly lower than the endurance

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<span id="page-1-0"></span>limit at  $10^7$  cycles [[2\]](#page-10-0). Thus, S-N curves of these steels show reasonably well-defined endurance limits under constant amplitude loading conditions.

Constant amplitude cycling below the endurance limit does not lead to fracture, however, it can cause fatigue damage. Several investigations show that short fatigue cracks may be initiated cycling carbon steels below the endurance limit [[3–6\]](#page-10-0). These cracks stop propagating when they reach a strong microstructural barrier, and the endurance limit is associated with a non-propagating condition of cracks rather than a non-initiating condition [\[3–6](#page-10-0)]. Under variable amplitude loading conditions, cyclic loads below the endurance limit can shorten the fatigue lifetime of steels  $[7-10]$ . The Miner rule  $[11, 12]$  $[11, 12]$  $[11, 12]$  $[11, 12]$  considers load cycles below the endurance limit non-damaging, and several adaptations are suggested to consider low load cycles in damage accumulation calculations. Predicted failures at damage sums below one [\[13](#page-10-0)] or different extrapolations of the S–N curve below the endurance limit for the purpose of damage accumulation calculations  $[14–16]$  $[14–16]$  are methods to consider detrimental influences of low load cycles. Low cyclic stress amplitudes can advance fatigue cracks in carbon steels, although they are not sufficiently high to propagate the crack to rupture [\[17](#page-10-0)]. Fracture mechanics tests show accelerated crack growth under the influence of numerous cycles below threshold stress intensity [\[18](#page-10-0)]. These influences of low load cycles on crack propagation are not considered appropriately in linear damage accumulation calculations and are possible reasons for the inaccuracy of the Miner rule.

Two-step variable amplitude (VA) tests are used in the present investigation to study the influences of low load cycles on fatigue damage in 0.15% C steel. Endurance tests are performed with repeat sequences consisting of few stress amplitudes above the endurance limit and numerous amplitudes below. Fracture mechanics two-step VA tests are performed with few stress intensity amplitudes above the threshold stress intensity and numerous amplitudes below, and the results of these testing series are compared to the respective constant amplitude tests. Relatively short repeat sequences are used in two-step VA tests to diminish sequence effects. The coaxing effect, for example, can lead to extended lifetimes and increased damage sums, if specimens are loaded with low amplitudes first and high amplitudes afterwards, whereas reduced lifetimes are found for high amplitude loading first and low amplitude loading afterwards [[19–22\]](#page-10-0).

Constant amplitude (CA) experiments serve to determine the endurance limit at very high limiting lifetimes and the threshold stress intensity at very low limiting crack growth rates. Cyclic tension–compression and cyclic torsion endurance tests are performed, and specimens are loaded up to  $10<sup>9</sup>$  cycles if they do not fail. Threshold stress intensity for fatigue crack growth under CA loading conditions is determined in fracture mechanics tests at limiting crack propagation rates below  $10^{-12}$  m/cycle. The high number of load cycles necessary to perform the experiments makes accelerated testing inevitable. Ultrasonic fatigue testing equipment working at cycling frequency of approximately 20 kHz is used to perform the experiments. The experimental procedure used to perform two-step variable amplitude experiments with ultrasonic equipment is described in detail.

## Material and method

## Material

Investigations are performed with 0.15% C steel (nomenclature according to European standard 10084 is C15E, material number is 1.1141). The chemical composition of the material is in  $%$  weight: 0.15% C, 0.20% Si, 0.45% Mn,  $\langle 0.035\% \text{ P}, \langle 0.035\% \text{ S} \rangle$  and rest Fe. Three different types of specimens are machined from rods with diameter  $\varnothing$ 20 mm, as shown in Fig. 1. Specimens with circular reduction of cross section in the centres are used to investigate the fatigue lifetime under cyclic tension–compression loading (Fig. 1a) and cyclic torsion loading (Fig. 1b), and single edge-notched specimens are used to perform fatigue crack growth studies (Fig. 1c). The surfaces of the specimens are grinded with abrasive paper of grade 1000. Some specimens are polished afterwards to obtain mirror like surface condition, which facilitates the detection of possibly initiated cracks. After preparation, the



Fig. 1 Specimen shapes used in the present investigation, a cyclic tension–compression endurance tests, b cyclic torsion endurance tests, and c fatigue crack growth tests

specimens are annealed in vacuum at 920  $\degree$ C for 45 min followed by furnace cooling to eliminate residual stresses. Ultimate tensile strength of the material is  $R_m =$ 418 ± 6 MPa, 0.2% proof stress is  $R_{p0.2} = 316 \pm 8$  MPa and fracture strain is  $A = 34 \pm 1\%$ . The microstructure of the material is about 20% pearlite and the rest ferrite with a mean ferrite grain size of 15  $\mu$ m.

#### Experimental procedure

Fatigue tests are performed with the ultrasonic fatigue testing method working at cycling frequency of about 20 kHz. The high testing frequency allows investigating the fatigue behaviour of materials at very high numbers of cycles and the fatigue crack propagation behaviour at very low growth rates within relatively short testing times. Five different testing series are conducted: (1) constant amplitude (CA) fatigue lifetime investigations under tension– compression loading, (2) CA fatigue lifetime investigation under cyclic torsion loading, (3) two-step variable amplitude (VA) fatigue lifetime investigations under tension– compression loading, (4) CA fatigue crack growth investigations, (5) two-step VA fatigue crack growth investigations. All tests are performed under fully reversed testing conditions (load ratio  $R = -1$ ) in ambient air of 18–22 °C and 40-60% relative humidity.

The equipment actually used in this investigation is specified in Ref. [[23\]](#page-10-0). Ultrasonic fatigue tests are performed stimulating specimens to resonance vibrations in the ultrasonic range, which causes cyclic straining and maximum strain amplitudes in the centres of the samples. Depending on the used setup and specimen geometry, cyclic tension–compression loading or cyclic torsion loading experiments may be performed, and fatigue lifetime or fatigue crack growth rates may be investigated. Cyclic load amplitude is controlled measuring the vibration amplitude of one specimen's end and controlling the amplitude in a closed-loop electronic circuit during the test. A calibration procedure with strain gauges serves to determine the linear dependence between the vibration amplitude of one specimen's end and the strain amplitude in specimen's centre, and stress amplitudes are calculated from the strain amplitudes using Hooke's law. A second closed-loop electronic circuit controls the cycling frequency and keeps the loading frequency at resonance frequency during the test. Accuracy of load control and frequency control is very high, i.e. the deviations between selected and actually measured vibration amplitudes are maximum  $\pm 1\%$  and cycling frequency and resonance frequency coincide within  $\pm 1$  Hz [[23\]](#page-10-0). More details about the ultrasonic fatigue testing method, experimental procedures to perform endurance and fatigue crack growth tests under cyclic tension–compression loading, cyclic torsion loading,

mixed mode loading or superimposed loading and the evaluation of results are described in Ref. [\[24](#page-10-0)].

Endurance tests are performed cycling the specimens at the selected stress amplitudes until fracture or until a fatigue crack of certain length is initiated in the specimen. Initiation and growth of fatigue cracks increases the specimen's compliance and reduces its resonance frequency. Monitoring of the cycling frequency serves to automatically stop endurance tests, when certain crack lengths are reached or when the specimens fail. In fatigue crack growth experiments, specimens are cycled at certain stress intensity amplitudes, and growth of the fatigue crack is observed at the polished specimen's surface. Video equipment and magnification lenses are used to obtain a resolution of crack length measurement of about 7 µm. The numbers of stress intensity cycles,  $\Delta N$  required for crack length increments,  $\Delta a$  of about 0.1 mm are evaluated to obtain one data point. The stress intensity amplitude,  $K_a$  is used to present crack growth data measured at load ratio  $R = -1$ . Calculation of the stress intensity amplitude for the specimen shown in Fig. [1](#page-1-0)c is described in Ref. [[24\]](#page-10-0).

Cyclic loading of specimens are not continuously, but they are loaded in series of pulses and cooling pauses (intermitted loading). Figure [2a](#page-3-0) shows the principle of a CA ultrasonic experiment. The selected pulse lengths are typically 50 ms (i.e. approximately 1000 cycles at the cycling frequency of approximately 20 kHz). After each pulse, cooling pauses of adequate length (typically 50- 500 ms) are chosen to prevent heating of the specimens. If necessary, forced-air cooling is used additionally to keep specimens' temperature below 25  $^{\circ}$ C. At the beginning of a pulse, the vibration amplitude increases within approximately 100-150 cycles to the selected amplitude. Then the vibration continues at the selected amplitude and after the pulse, the vibration amplitude decreases to zero. Figure [2](#page-3-0)b shows the measured cumulative frequency distribution of load cycles of a CA endurance test. The specimen was loaded with 3208 pulses of 50 ms length, and the resonance frequency was 20.40 kHz. If all load amplitudes would be at the selected amplitude,  $3.27 \times 10^6$  cycles at stress amplitude 290 MPa would have been measured. Due to the increase of the vibration amplitude at the beginning of the pulse, however, solely  $2.97 \times 10^6$  cycles are greater than 275 MPa (i.e. greater than 95% of the selected stress amplitude).

Ultrasonic VA experiments are performed varying the vibration amplitude of successive pulses. In the present investigation, two-step VA tests serve to investigate the influences of stress cycles below the endurance limit on fatigue lifetime and the influences of stress intensity cycles below the threshold stress intensity on the crack propagation rate, respectively. Figure [3a](#page-4-0) shows the principle of a two-step VA experiment. The repeat sequence is one pulse

<span id="page-3-0"></span>Fig. 2 Constant amplitude tests: a principle of specimen's loading in series of pulses and pauses, b specified and measured cumulative frequency distribution of load cycles in a constant amplitude test at stress amplitude  $\sigma_a = 290$  MPa



of 50 ms at the high amplitude and pre-selected number of pulses of 50 ms each at the low amplitude. High amplitudes are above endurance limit or threshold stress intensity and low amplitudes are below. Figure [3b](#page-4-0) shows the measured cumulative frequency distribution of load cycles of a twostep VA endurance experiment. The specimen was loaded with a repeat sequence consisting of 1 pulse of 50 ms at stress amplitude 290 MPa and 100 pulses of 50 ms at stress amplitude 240 MPa, and this repeat sequence was repeated 222 times. With the resonance frequency of 20.40 kHz,  $2.26 \times 10^7$  load cycles at 290 MPa would be attended (specified distribution in Fig. [3](#page-4-0)b). The measured distribution of load amplitudes shows  $2.06 \times 10^7$  load cycles greater than 275 MPa (i.e. greater than 95% of the high stress amplitude). Load amplitudes increase at the beginning of a pulse and thus approximately 90% instead of 100% are at the specified amplitude. Damage accumulation calculations are performed with the actually measured and not with the theoretical distribution of load amplitudes.

Two-step VA endurance experiments are performed with four different high stress amplitudes,  $\sigma_{\text{high}}$ . The low stress amplitude  $\sigma_{\text{low}}$  was kept constant at 240 MPa in all two-step VA endurance tests. It will be shown that 240 MPa is below the endurance limit of the investigated 0.15% C steel. The repeat sequences in two-step VA endurance tests are shown in Table. [1.](#page-4-0) Minimum six samples are cycled until failure or until total of  $2 \times 10^9$  cycles (at high and low amplitude) are reached. The measured lifetime is compared with the predicted lifetime using linear damage accumulation calculation (Miner calculation). Two-step VA fatigue crack growth tests are performed with five different high stress intensity amplitudes,  $K_{\text{high}}$ . The low stress intensity  $K_{\text{low}}$ was kept constant at  $3.4$  MPa m<sup>1/2</sup> in all tests. It will be shown that 3.4 MPa  $m^{1/2}$  is below the threshold stress intensity of the investigated 0.15% C steel. The repeat sequences in two-step VA fatigue crack growth tests are shown in Table [2.](#page-4-0)

## Results

# Fatigue lifetime under CA tension–compression loading

Constant amplitude (CA) endurance data of C15E measured under cyclic tension–compression loading are shown in Fig. [4a](#page-5-0). Arrows indicate measurements, where the specimen did not fail until minimum  $10<sup>9</sup>$  cycles are reached. Two approximation lines are visible, which indicate 50% fracture probability. For stress amplitudes with more than 50% failures, (i.e. stress amplitude 270 MPa or higher), the approximation line is calculated assuming

 $(a)$ 

High Amplitude

Low Amplitude

 $(b)$ 

 $10^{10}\,$ 

 $10<sup>9</sup>$ 

<span id="page-4-0"></span>Fig. 3 Two-step variable amplitude tests: a principle of specimen loading with repeat sequence consisting of one pulse at the high amplitude and pre-selected number of pulses at the low amplitude, the high amplitudes are above the endurance limit (in endurance tests) or above the threshold stress intensity (in fatigue crack growth tests), and the low amplitudes are below, b specified and measured cumulative frequency distribution of load cycles in a two-step variable amplitude tests with high stress amplitude  $\sigma_{\text{high}} = 290$  MPa and low stress amplitude  $\sigma_{\text{low}} = 240 \text{ MPa}$ 





**Specified Distribution** 

Test series	High stress amplitude, $\sigma_{\text{high}}$ (MPa)	Numbers of cycles at $\sigma_{\text{high}}$	Low stress amplitude, $\sigma_{\text{low}}$ (MPa)	Numbers of cycles at $\sigma_{\text{low}}$	Predicted total number of cycles to failure
	330	$10^3$	240	$400 \times 10^{3}$	$3.2 \times 10^8$
2	310	10 <sup>3</sup>	240	$200 \times 10^{3}$	$3.2 \times 10^{8}$
3	290	$10^3$	240	$100 \times 10^{3}$	$3.2 \times 10^{8}$
4	270	$10^3$	240	$45 \times 10^{3}$	$3.2 \times 10^8$

Table 2 Repeat sequences of two-step VA fatigue crack growth tests

Table 1 Repeat sequences of two-step VA endurance tests



power law dependence between stress amplitudes and cycles to failure and normal distribution of the logarithms of fatigue lifetimes. Mean lifetime of  $10^7$  cycles (i.e. 50%) fracture probability below  $10<sup>7</sup>$  cycles) is at stress amplitude  $\sigma_a = 266 \pm 16$  MPa. The mean endurance limit at limiting lifetime of 10<sup>9</sup> cycles (i.e. 50% fracture probability below  $10^9$  cycles) is  $\sigma_{en} = 261 \pm 16$  MPa. The straight line approximating data in the HCF and VHCF regime shows a small slope due to failures that occurred in the regime between  $10^7$  and  $10^9$  cycles.

<span id="page-5-0"></span>

Fig. 4 Constant amplitude endurance data, a cyclic tension–compression loading, b cyclic torsion loading

Two-step variable amplitude endurance tests are performed with the low stress amplitude  $\sigma_{\text{low}} = 240$  MPa. The calculated probability for failure below  $10<sup>9</sup>$  cycles at 240 MPa is 7%, and all seven specimens tested at constant stress amplitude 240 MPa survived minimum  $10<sup>9</sup>$  cycles.

#### Fatigue lifetime under CA cyclic torsion loading

Endurance data measured in ultrasonic cyclic torsion tests are shown in Fig. 4b. Statistical evaluation and approximation of torsion fatigue data using two approximation lines are performed similar to the statistical evaluation of tension–compression data described above. Mean lifetime of  $10^7$  cycles is found at torsion stress amplitude  $\tau_a = 164 \pm 16$  MPa. The mean endurance limit at limiting lifetime 10<sup>9</sup> cycles is  $\tau_{en} = 154 \pm 14$  MPa. The ratio of mean endurance limits at  $10^9$  cycles for cyclic torsion



Fig. 5 Non-propagating fatigue crack visible on the surface of a specimen after cyclic torsion loading at  $\tau_a = 159$  MPa, no failure after  $1.0 \times 10^9$  cycles, specimen's length direction is from top to bottom

loading and cyclic tension–compression loading is by mean  $\tau_{en}/\sigma_{en} = 0.59$ . This quotient is close to  $(1/3)^{1/2}$  suggested by the Mises equivalent stress hypothesis. Ratios of  $\tau_{en}$  and  $\sigma_{en}$  published in the literature for steel are in the range from 0.58 and 0.72 [[17,](#page-10-0) [25–30\]](#page-10-0).

The surfaces of specimens, which survived the cyclic torsion test without fracture, are investigated in the scanning electron microscope using a rotating stage. Figure 5 shows a section of the surface of a specimen that survived  $10<sup>9</sup>$  cycles without failure. A ferrite grain with strong slip activity is visible, which produces a rough surface. Slip lines are orientated approximately in specimen's length direction, which is one direction of maximum shear stress. Where slip bands encounter the grain boundary, a short fatigue crack is formed at the grain boundary. Obviously, this crack did not propagate to fracture during  $10<sup>9</sup>$  cycles and may be considered as non-propagating crack. In some specimens that survived  $10^9$  cycles, short cracks at intersections of slip lines and grain boundaries were similarly visible whereas in other specimens, no cracks could be found. These observations suggest that the endurance limit in this carbon steel is the maximum stress amplitude, which is not sufficient to propagate possibly existing cracks to failure. This supports the assumption  $[3-6]$  that the endurance limit in carbon steel is determined by a nonpropagating condition of cracks rather than a non-initiating condition.

Fatigue lifetime under two-step VA tension– compression loading

Four series of two-step variable amplitude (VA) endurance tests are performed with repeat sequences shown in Table [1](#page-4-0). The results of these tests are presented in Figs. [6,](#page-6-0) [7](#page-6-0) and [8](#page-6-0). In Fig. [6,](#page-6-0) two-step VA endurance data are

<span id="page-6-0"></span>

Fig. 6 Endurance data of two-step variable amplitude tests presented versus the numbers of cycles at the high stress amplitude (closed circles), constant amplitude data are included for comparison (open circles)



Fig. 7 Endurance data of two-step variable amplitude tests presented versus the total numbers of load cycles, the predicted lifetimes using linear damage accumulation calculation (Miner calculation) and damage sum 1 is  $3.2 \times 10^8$  cycles in all tests

presented versus the high stress amplitude,  $\sigma_{\text{high}}$ . The abscissa shows the number of cycles at  $\sigma_{\text{high}}$  until failure or until the experiment is stopped (data of unbroken specimens are marked with arrows). For comparison, CA data are included in Fig. 6. In Fig. 7, two-step VA endurance data are presented versus the total number of cycles, i.e. the sum of high and low amplitude cycles. Additionally, the expected lifetime performing a linear damage accumulation calculation (Miner calculation) and damage sum 1 (which is  $3.2 \times 10^8$  cycles in all four testing series) is indicated. Figure 8 presents the damage sum of each



Fig. 8 Damage sums in linear damage accumulation calculations

specimen versus the high stress amplitude of the twostep VA test. Additionally, the mean endurance limit at  $10^9$  cycles (i.e.  $\sigma_{en} = 261 \pm 16$  MPa) is indicated.

Two-step VA tests with  $\sigma_{\text{high}} = 330$  MPa show failures after total numbers of cycles ranging from  $9.6 \times 10^6$  to  $3.1 \times 10^8$  (Fig. 7). This means that the specimens failed after minimum 2.4  $\times$  10<sup>4</sup> cycles and maximum 7.8  $\times$  $10<sup>5</sup>$  cycles at the high stress amplitude (Fig. 6). Statistical evaluation of S–N data delivers mean lifetime of 7.8  $\times$  10<sup>5</sup> cycles for CA loading at 330 MPa, whereas the mean number of cycles at the high amplitude until failure is  $1.9 \times 10^5$  cycles in VA tests. The damage sum of the seven investigated specimens varies between 0.03 for the weakest specimen and 0.98 for the strongest (Fig. 8), and the geometric mean of the damage sum is 0.24. These results demonstrate the detrimental influences of low load cycles leading to shortened fatigue lifetimes for VA loading with  $\sigma_{\text{high}} = 330$  MPa.

In two-step VA tests with  $\sigma_{\text{high}} = 270$  MPa, all tested specimens survived more than 3.6  $\times$  10<sup>9</sup> cycles at the high and low stress amplitude without failure (Fig. 7). This means that these specimens survived more than 7.8  $\times$  10<sup>7</sup> cycles at  $\sigma_{\text{high}} = 270$  MPa (Fig. 6). The mean lifetime for CA loading at 270 MPa is  $7.0 \times 10^6$  cycles, and six of the seven specimens tested in the CA experiment failed below maximum  $3.1 \times 10^7$  cycles. Thus, all specimens cycled in the two-step VA test with  $\sigma_{\text{high}} = 270 \text{ MPa}$ survived damage sums greater than 10 without failure (Fig. 8). These results indicate beneficial rather than detrimental influences of low load cycles leading to extended fatigue lifetime for VA loading with  $\sigma_{\text{high}} = 270$  MPa.

Beneficial influences of low amplitude cycles on the fatigue properties are similarly found in VA endurance tests with the high stress amplitude  $\sigma_{\text{high}} = 290 \text{ MPa}$ . Six

specimens were tested and five specimens survived minimum 2.2  $\times$  10<sup>9</sup> cycles (Fig. [7\)](#page-6-0), which means that they survived minimum 2.2  $\times$  10<sup>7</sup> cycles at  $\sigma_{\text{high}} = 290 \text{ MPa}$ (Fig. [6](#page-6-0)). One specimen failed after  $8.3 \times 10^6$  cycles at  $\sigma_{\text{high}} = 290$  MPa. In CA tests at 290 MPa, all seven tested specimens failed below maximum  $5.5 \times 10^6$  cycles, and the mean CA lifetime at this stress amplitude is  $3.2 \times 10^6$  cycles. This means that five out of six specimens tested under two-step VA loading with  $\sigma_{\text{high}} = 290 \text{ MPa}$ survived damage sums greater than 6.7 and one specimen failed at damage sum 2.6 (Fig. [8](#page-6-0)). Similar to the experiments with  $\sigma_{\text{high}} = 270 \text{ MPa}$ , specimens can withstand larger numbers of high stress amplitudes without failure, when they were additionally cycled with numerous stress amplitudes below the endurance limit.

In two-step VA experiments with  $\sigma_{\text{high}} = 310 \text{ MPa}$ , detrimental influences of low load cycles were found in four specimens. They failed after loading with total  $1.3 \times 10^{7}$ - $7.0 \times 10^{7}$  $7.0 \times 10^{7}$  cycles (Fig. 7), which means that they failed after  $6.7 \times 10^4 - 3.5 \times 10^5$  $6.7 \times 10^4 - 3.5 \times 10^5$  cycles at 310 MPa (Fig. 6). In CA tests at stress amplitude 310 MPa, all specimens survived minimum  $6.4 \times 10^5$  cycles. Damage sum of these four specimens are in the regime from 0.04 to 0.21. In contrast, two specimens survived minimum  $2.6 \times$  $10^9$  cycles (1.3  $\times$  10<sup>7</sup> cycles at 270 MPa), which means that the damage sums of these specimens are greater than 8. Experiments with  $\sigma_{\text{high}} = 310 \text{ MPa}$  show no univocal result, but detrimental influences of low load cycles are found in some specimens and beneficial in others.

In Fig. [6](#page-6-0), solid lines indicate 50% fracture probability in two-step VA tests considering solely high amplitude cycles. Fifty percent fracture probability at  $10<sup>7</sup>$  cycles is obtained for high stress amplitude  $301 \pm 20$  MPa. In CA tests, mean lifetime of  $10<sup>7</sup>$  cycles is determined at stress amplitude  $266 \pm 16$  MPa. The material can sustain about 13% higher stress amplitudes without failure, if it is additionally loaded with load cycles below the endurance limit. If the high stress amplitude of the VA is more than 13% above the CA endurance limit at  $10^9$  cycles, detrimental influences of low load cycles prevail. Beneficial influences of low load cycles prevail, if the high stress amplitude is less than 13% above the CA endurance limit.



Figure 9 shows the results of CA fatigue crack growth tests at load ratio  $R = -1$ . Crack growth rates,  $\Delta a / \Delta N$  are presented versus the stress intensity amplitude,  $K_a$ . Fatigue crack growth tests are performed to investigate the lower Paris regime and to determine the threshold stress intensity. Typical crack growth curve is visible with a pronounced change of slope at growth rates of about  $10^{-10}$  m/cycle.



Fig. 9 Fatigue crack growth under fully reversed cyclic tension– compression loading

Crack growth data with growth rates above  $10^{-10}$ m/cycle approximately follow a straight line in this double logarithmic plot, as suggested by the Paris law. The Paris exponent determined for the investigated material and load ratio  $R = -1$  is 3.4. Arrows indicate measurements, where no crack growth was visible within minimum  $2 \times 10^7$  cycles within the optical resolution of 7 µm. Crack growth and threshold investigations were performed with different specimens and evaluating crack propagation rates at different crack lengths. The highest stress intensity amplitude, where no crack growth could be detected within  $2 \times 10^7$  cycles, was 3.85 MPa m<sup>1/2</sup>, and the lowest stress intensity amplitude with visible crack growth was 3.55 MPa  $m^{1/2}$ . The threshold stress intensity of C15E is considered in the regime  $K_{\text{th}} = 3.55 - 3.85 \text{ MPa m}^{1/2}$ . In all two-step VA fatigue crack growth tests the low stress intensity amplitude was  $K_{\text{low}} = 3.40 \text{ MPa m}^{1/2}$ .  $K_{\text{low}}$  is below  $K_{\text{th}}$ , and no crack growth may be expected cycling the material at this cyclic load with constant amplitude.

Fatigue crack propagation under two-step VA tension– compression loading

Repeat sequences used to perform two-step VA fatigue crack growth tests are summarized in Table [2](#page-4-0). Figure [10](#page-8-0) shows the results of an experiment performed with  $K_{\text{high}} = 5.5 \text{ MPa m}^{1/2}$ . The specimen is cycled at constant stress intensity amplitude 5.5 MPa  $m^{1/2}$  prior to two-step VA loading at crack length between 2.8 and 3.5 mm and after two-step VA loading at crack lengths between 4.3 and 4.8 mm. This serves to exactly determine the crack growth

<span id="page-8-0"></span>

Fig. 10 Mean crack propagation rates for constant amplitude loading  $(K_a = 5.5 \text{ MPa m}^{1/2})$ , crack lengths from 2.8 to 3.5 mm and from 4.3 to 4.8 mm) and for two-step VA loading ( $K_{\text{high}} = 5.5$  MPa m<sup>1/2</sup> and  $K_{\text{low}} = 3.4 \text{ MPa m}^{1/2}$ , crack length from 3.5 to 4.3 mm), mean growth rates for constant amplitude loading (dashed line), two-step VA loading (solid line) and calculated growth rate assuming no influence of low load cycles (dash dotted line) are shown

rate for CA loading. Mean growth rate cycling at constant stress intensity amplitude 5.5 MPa m<sup>1/2</sup> is  $6.0 \times 10^{-10}$ m/cycle, which is indicated with dashed lines in Fig. 10. At crack length between 3.5 and 4.3 mm, the specimen is loaded with the two-step VA sequence. If crack propagation in two-step VA loading is solely caused by high stress intensity cycles and low stress intensity cycles have no influence, the mean crack propagation rate would be  $6.0 \times 10^{-12}$  m/cycle, which is shown as dash-dotted line in Fig. 10. The mean growth rate actually found in the twostep VA sequence is  $1.5 \times 10^{-11}$  m/cycle, which is indicated as solid line. The ratio of the experimentally found propagation rate in VA test and the predicted growth rate using constant amplitude data is named ''acceleration ratio'' [[18\]](#page-10-0). In the two-step VA sequence shown in Fig. 10, the acceleration ratio is 2.5. Figure 11 shows the acceleration ratios determined in 10 two-step VA fatigue crack growth tests. When the high stress intensity  $K_{\text{high}}$  is in the range from 4.35 to 5.5 MPa  $m^{1/2}$ , the acceleration ratio is between 1.5 and 3.1 (mean value 2.2). For these high stress intensities, cyclic loading below threshold stress intensity accelerates crack growth.

Two-step VA experiments with  $K_{\text{high}} = 4.2 \text{ MPa m}^{1/2}$ lead to different results. Cycling the specimen at constant stress intensity amplitude 4.2 MPa  $m^{1/2}$  prior to two-step VA loading, mean crack propagation rates of  $2.5 \times 10^{-10}$  m/cycle are found. When the specimen is cycled with two-step VA loading afterwards, no further crack growth can be observed, although the specimen is loaded with total  $2 \times 10^8$  cycles. Within the optical resolution of 7 lm, possible mean crack growth rates are below



Fig. 11 Acceleration ratios versus the high stress intensities in twostep variable amplitude fatigue crack growth tests

 $3.5 \times 10^{-14}$  m/cycle, which is far below the expected growth rate of 2.5  $\times$  10<sup>-12</sup> m/cycle assuming no influence of low amplitude cycles. Acceleration ratios found in the two experiments with the high stress intensity 4.2 MPa  $m^{1/2}$ are below 0.014, which is indicated with arrows in Fig. 11.

Threshold stress intensity under CA loading conditions is 3.55–3.85 MPa  $m^{1/2}$ , whereas stop of fatigue crack growth was determined in two-step VA experiments at  $K_{\text{high}} = 4.2 \text{ MPa m}^{1/2}$ . These experiments show beneficial influences and strong retardation of fatigue crack growth caused by low load cycles, if the high stress amplitude of the VA is 14% below the CA threshold stress intensity. In experiments with the high load 18% or more above the CA threshold stress intensity, detrimental influences of low load cycles and accelerated crack growth are found.

#### **Discussion**

Two-step VA endurance tests demonstrate that low amplitude cycles can have detrimental or beneficial influences on fatigue damage in C15E leading to shortened or prolonged fatigue lifetimes, respectively. If the maximum stress amplitude of the VA repeat sequence is more than 13% above the mean endurance limit at  $10^9$  cycles, detrimental influences of low load cycles prevail, whereas beneficial influences and damage sums far beyond one could be found for lower maximum stress amplitudes. Influence of low load cycles on fatigue lifetime can be correlated to VA fatigue crack growth, where low amplitude cycles caused accelerated or retarded fatigue crack propagation depending on the maximum amplitude of the VA load sequence.

Shortened fatigue lifetimes of steels due to load amplitudes below the endurance limit are well documented in the literature [[7](#page-10-0)–[10\]](#page-10-0). Crack propagation of short cracks [[17\]](#page-10-0) and long cracks [[18\]](#page-10-0) may be accelerated by load amplitudes, which are too low to propagate the crack to rupture. Mean acceleration ratio determined in the present investigation is 2.2, however, acceleration ratios may reach up to 100 for certain conditions of the high and low load amplitudes [\[18](#page-10-0)]. In a previous study performing two-step VA tests with cast aluminium alloy 319-T7 [[31\]](#page-10-0), mean acceleration ratio of 3.6 was found when the high stress intensity amplitude was more than 15% above threshold stress intensity. These investigations demonstrate that numerous load cycles below the endurance limit can accelerate fatigue crack propagation. Consequently, reduced fatigue lifetimes of C15E and damage sums significantly below one result, if the maximum load of the VA sequence is more than 13% above the endurance limit.

Beneficial influences of low load cycles under VA loading conditions are rarely described in the literature. When beneficial influences are reported, they are related to the coaxing effect in the majority of cases [[19–22](#page-10-0)], i.e. extended fatigue lifetimes and damage sums greater than 1 are found when specimens are cycled with low stress amplitudes first and high amplitudes afterwards. In the present investigation, however, relatively short repeat sequences are used. The repeat sequence is repeated minimum 80000 times in two-step VA endurance tests with  $\sigma_{\text{high}} = 290 \text{ MPa}$  and  $\sigma_{\text{high}} = 270 \text{ MPa}$ , where beneficial influences of low load cycles are most obvious. The load amplitude frequently changes between high and low stress amplitudes, and the coaxing effect cannot explain the beneficial influences of low load cycles found in the present two-step VA tests therefore. Constant amplitude tests, albeit under cyclic torsion loading, show that fatigue cracks can be initiated cycling the material at the endurance limit, and the endurance limit is considered as maximum stress amplitude that cannot propagate possibly initiated cracks to fracture. Two-step VA fatigue crack growth tests show that crack propagation can stop at stress intensity amplitudes 14% above the CA threshold under the influence of numerous low load amplitudes. Similarly, twostep VA loading of cast aluminium alloy 319-T7 showed stop of crack growth when the high stress amplitude was less than 15% above threshold stress intensity [\[31](#page-10-0)]. In the wrought aluminium alloy 6261-T6, pronounced retardation of short fatigue crack growth is reported, if the high stress amplitudes in two-step VA tests are only slightly higher than the low stress amplitudes [[32\]](#page-10-0). Numerous low amplitude cycles cause higher maximum stress intensity amplitudes in VA sequences that are necessary to advance a fatigue crack. Consequently, higher stress amplitudes are necessary under VA than under CA loading conditions to propagate a crack to fracture. The material can withstand higher stress amplitudes without failure when it is additionally loaded with numerous low load cycles, and extended lifetimes and high damage sums without failure result.

## Conclusions

Fatigue behaviour of low carbon steel C15E is investigated at load ratio  $R = -1$  using ultrasonic fatigue testing equipment. Constant amplitude (CA) tests are performed to determine the endurance limit at very high numbers of cycles and the threshold stress intensity at very low limiting crack growth rates, respectively. Two-step variable amplitude (VA) tests are conducted to investigate the influences of load cycles below the endurance limit on fatigue lifetime and the influences of load cycles below the threshold stress intensity on crack propagation. The following results are obtained.

- 1. CA endurance limits at limiting lifetime of  $10<sup>9</sup>$  cycles are determined under cyclic tension–compression and cyclic torsion loading. The ratio of cyclic torsion and cyclic tension–compression endurance limit is by mean  $\tau_{en}/\sigma_{en} = 0.59$ . Non-propagating fatigue cracks are found in specimens subjected to cyclic torsion loading at the endurance limit. The endurance limit is considered as maximum stress amplitude where possibly initiated fatigue cracks do not propagate to failure.
- 2. Two-step VA endurance and fatigue crack growth experiments can show detrimental or beneficial influences of low load cycles on fatigue damage, and the maximum load amplitude of the VA repeat sequence determines which effect prevails. If the high stress amplitude is more than 13% above the endurance limit, low stress cycles shorten the fatigue lifetime and linear damage accumulation calculations deliver damage sums below 1. Two-step VA fatigue crack growth studies show that low amplitude cycles can accelerate fatigue crack propagation under these conditions for the high load, which consequently reduces the fatigue lifetime.
- 3. If the high stress amplitude is less than 13% above the endurance limit, numerous low stress cycles prolong the fatigue lifetime. Damage sums greater than 10 can be found without failure. Under these conditions for the high load, numerous low amplitude cycles can stop fatigue crack growth. Consequently, the material can withstand higher stress amplitudes without failure when it is additionally loaded with low load cycles.

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